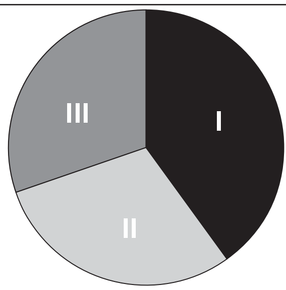
THE **PRAXIS** SERIES

Mathematics: Proofs, Models, and Problems, Part 1 (0063)

Test at a Glance

Test Name	**Mathematics: Proofs, Models, and Problems, Part 1**		
Test Code	**0063**		
Time	**1 hour**		
Number of Questions	**4 basic exercises: 1 proof, 1 model, and 2 problems**		
Format	**Constructed-response questions, graphing calculator required**		
	Content Categories	Approximate Number of Questions	Approximate Percentage of Examination
	I. Problems	2	40%
	II. Model	1	30%
	III. Proof	1	30%
	Process Categories		
	Mathematical Problem Solving Mathematical Reasoning and Proof Mathematical Connections Mathematical Representation Use of Technology		Distributed Across Content Categories

About This Test

The Mathematics: Proofs, Models, and Problems, Part 1, test is designed to assess the mathematical knowledge and competencies necessary for a beginning teacher of secondary school mathematics. Examinees have typically completed a bachelor's program with an emphasis in mathematics or mathematics education.

The test is not designed to be aligned with any particular school mathematics curriculum, but it is intended to be consistent with the recommendations of recent national studies on mathematics education such as the National Council of Teachers of Mathematics (NCTM) *Principles and Standards for School Mathematics* (2000) and the National Council for Accreditation of Teacher Education (NCATE) *Program Standards for Initial Preparation of Mathematics Teachers* (2003).

This mathematics test requires an examinee to demonstrate an understanding of basic mathematical concepts and their applications in constructing a model, writing a proof, and solving two problems. The test assesses knowledge in the following five content areas: algebra and number theory; measurement, geometry and trigonometry; functions; data analysis, statistics (without calculus), and probability; and matrix algebra and discrete mathematics. The test focuses on problem solving, reasoning and proof, and mathematical connections and representations. (For a description of the mathematics and process content, see Mathematics: Content Knowledge Test 0061.) Competencies from more than one content area may be required in the course of answering any question.

Each problem is worth 20 percent of the total test score. Collectively, the two problems represent approximately 40 percent of the total test score, and the model and the proof questions each represent approximately 30 percent of the total test score.

The four exercises require the ability to understand and work with mathematical concepts, to reason mathematically, to integrate knowledge of different areas of mathematics, and to develop mathematical models of real-life situations. Constructing a response to each exercise demonstrates the ability to present a solution to a mathematical problem or to explain a mathematical idea in a manner that is correct, complete, clear, and coherent.

Graphing calculators without QWERTY (typewriter) keyboards are required for this test. For a description of the minimum capabilities required of the calculator, see the Graphing Calculator section of Content Knowledge Test 0061 on page 2.

Selected notations, formulas, and definitions are printed in the test book and are also listed on pages 8–10 of the Content Knowledge Test 0061 information.

Using the Graphing Calculator

If you use a graphing calculator in answering a question, your response must include a mathematical description of what you use the calculator to do rather than a record of the keystrokes you use. When you use the graphing calculator, keep in mind that the graph produced in the viewing window may not give sufficient information to answer the question because of such factors as screen resolution and the fact that any part of the graph of the function beyond the viewing window is not shown (e.g., the graph of the function $y = x^2 + 10x - 11$ on the viewing window $[-10, 10] \times [-10, 10]$ looks like a line instead of a parabola, and the graph of the function $y = \sin \frac{1}{x}$ on the viewing window $[-10, 10] \times [-10, 10]$ will not show the true behavior of the function around $x = 0$).

Sample Test Questions

This section presents sample questions and constructed-response samples along with the standards used in scoring the basic exercises in proofs, models, and problems. When you read these sample responses, keep in mind that they will be less polished than if they had been developed at home, edited, and carefully presented. Examinees do not know what questions will be asked and must decide, on the spot, how to respond. Readers take these circumstances into account when scoring the responses.

Readers will assign scores based on the following scoring guide.

SCORING GUIDE

5

- Clearly demonstrates a full understanding of the mathematical content necessary to answer all parts of the question successfully
- Gives a correct and complete response but may contain a minor calculation error

4

- Clearly demonstrates a full understanding of the mathematical content needed to answer all parts of the question successfully
- *Either* gives a complete response that contains a minor mathematical error or misstatement
  OR gives a correct and almost complete response

3

For a one-part question:

- Clearly demonstrates an understanding of all aspects of the question
- Demonstrates the ability to determine an appropriate strategy for answering the question
- Makes substantial progress toward a correct and complete response

For a multipart question:

- Clearly demonstrates a full understanding of the mathematical content needed to answer a significant portion of the question successfully
- Gives a correct and complete response to that portion of the question

2

For a one-part question:

- *Either* demonstrates a limited understanding of the question
  OR makes only minimal progress toward a correct and complete response

For a multipart question:

- Clearly demonstrates a full understanding of the mathematical content needed to answer a minor portion of the question successfully
- Gives a correct and complete response to that portion of the question

1

- Demonstrates a very limited understanding of the question
- Makes little or no progress toward a correct and complete response

0

- Blank, almost blank, or off topic

Sample Question 1: Problems

$\frac{1}{1} = 1.0$ $\qquad \frac{1}{4} = 1.0$

$\frac{1}{2} = 0.5$ $\qquad \frac{1}{5} = 0.20$

$\frac{1}{3} = 0.3$ $\qquad \frac{1}{6} = 0.1\overline{6}$

Is it true that for any positive integer n, the decimal expansion of $\frac{1}{n}$ either terminates or begins a repeating pattern within at most $n - 1$ digits to the right of the decimal point? Explain why or why not.

Sample Response That Received a Score of 5:

$$\frac{1}{7} : \begin{array}{r} .1428571428\ldots \\ 7\overline{)1.000000} \\ \underline{7} \\ 30 \\ \underline{28} \\ 20 \\ \underline{14} \\ 60 \\ \underline{56} \\ 40 \\ \underline{35} \\ 50 \\ \underline{49} \\ 10 \\ \underline{7} \\ 30 \\ \underline{28} \\ 20 \\ \underline{14} \\ 60 \end{array} = .\overline{142857}$$

$$\frac{1}{8} : \begin{array}{r} .125 \\ 8\overline{)1.0000} \\ \underline{8} \\ 20 \\ \underline{16} \\ 40 \\ \underline{40} \\ 0 \end{array} = 0.125$$

$$\frac{1}{9} : \begin{array}{r} .111\ldots \\ 9\overline{)1.0000} \\ \underline{9} \\ 10 \\ \underline{9} \\ 10 \end{array} = 0.\overline{1}$$

$$\frac{1}{10} : \begin{array}{r} .1 \\ 10\overline{)1.0000} \\ \underline{1\ 0} \\ 0 \end{array}$$

In each step of the division there is a remainder. For $\frac{1}{7}$ the only possible remainders are 0, 1, 2, 3, 4, 5, 6. Once a remainder repeats in the process the whole pattern repeats. There is nothing special about 7; it works with n also except the number of remainders is n. In the examples above each number either terminates when the remainder is 0 or repeats within $(n - 1)$ steps of the division process; therefore, the statement is true.

Sample Response That Received a Score of 2:

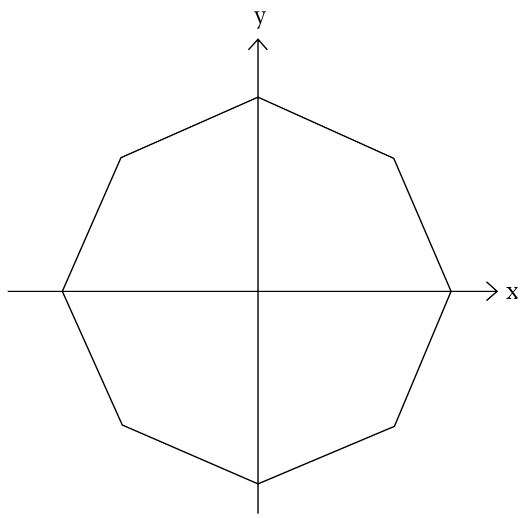
Yes, it is true. If n is a prime, then $\frac{1}{n}$ will repeat somewhere in the expansion. Also, if n has any prime as a factor, $\frac{1}{n}$ will repeat.

Also, if 3 is a factor of n, $\frac{1}{n}$ will repeat somewhere in the expansion since $\frac{1}{3}$ repeats.

If n has only 2's or 5's as factors, then $\frac{1}{n}$ terminates.

In fact, this is the only way $\frac{1}{n}$ will terminate.

Sample Question 2: Problems



In the figure above, the center of the regular octagon is at the origin.

(A) For how many angles θ, $0 \le \theta < 2\pi$, will a rotation of the octagon by an angle θ map the octagon onto itself? Explain how you arrived at your answer.

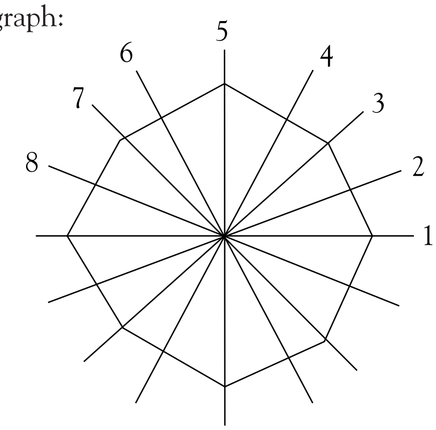
(B) How many reflections of the octagon will map the octagon onto itself? Explain how you arrived at your answer.

Sample Response That Received a Score of 4:

(A) For eight angles θ, $0 \le \theta < 2\pi$, will a rotation of the octagon by angle θ map the octagon onto itself. I arrived at this answer by observation of the graph. The eight angles are as follows:

$0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}.$

(B) Eight reflections of the octagon will map the octagon onto itself, one about each of the axes in my graph:



Sample Response That Received a Score of 2:

(A) By rotating the angle $0 \le x \le \frac{\pi}{2}$ four times from Quadrants 1-4 respectively you will have a map of the octagon onto itself.

(B) Because the octagon is symmetrical it can be reflected across the x-axis ($0 \le \theta < \pi$), across the y-axis ($0 \le \theta < \frac{\pi}{2}$). It also can be reflected from Quadrant I to Quadrant III ($0 \le \theta < \frac{\pi}{2}$) and from Quadrant II to Quadrant IV ($\frac{3\pi}{2} \le \theta < 2\pi$).

Sample Question 3: Problems

Explain why division by zero is undefined.

Sample Response That Received a Score of 5:

If we were to define division by 0, the definition would have to be consistent with how we define division by any other number. Division is defined on the basis of multiplication.

If $\frac{a}{b} = c$, then $a = b \cdot c$.

So if $\frac{a}{0} = c$, then $a = c \cdot 0$, so $a = 0$ and no other number a is possible.

But $0 \cdot$ anything $= 0$, so we can't find a value for c. This means we can't divide any number by 0.

Sample Response That Received a Score of 1:

$\frac{a}{0} = 0$ but $0 \cdot 0 = a$ is not right.

Sample Question 4: Problems

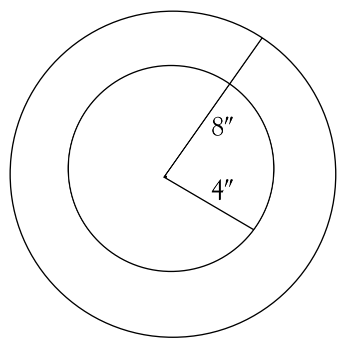
A gnat is randomly flying inside a spherical balloon of radius 8 inches. What is the probability that at any given time the gnat is at a distance of more than 4 inches from the center of the balloon? Show how you determined the probability.

Sample Response That Received a Score of 5:

$$P = \frac{V(\text{Large Sphere}) - V(\text{Little Sphere})}{\text{Volume (Balloon)}}$$

$$P = \frac{\frac{4}{3}\pi 8^3 - \frac{4}{3}\pi 4^3}{\frac{4}{3}\pi 8^3}$$

$$= \frac{\frac{4}{3}\pi(8^3 - 4^3)}{\frac{4}{3}\pi 8^3} = \frac{8^3 - 4^3}{8^3} = \frac{512 - 64}{512}$$



$$= \frac{448}{512} = \frac{7}{8}$$

Sample Response That Received a Score of 1:

$$\frac{V(\text{outside } \frac{1}{2} \text{ of balloon})}{\frac{4}{3}\pi(R^3)} = \frac{\frac{1}{2}\pi(4^3)}{\frac{4}{3}\pi(8^3)}$$

$$\frac{32\pi}{\frac{4}{3}\pi(512)} = \frac{24}{512} = \frac{3}{64}$$

Sample Question 5: Problems

Consider the function $f(x) = \sqrt{4 - x^2}$

(A) What is the largest interval on which $f(x)$ is real-valued?

(B) What is the largest interval containing -1 for which the function $f^{-1}(x)$ exists?

(C) Give a formula for the inverse function for f restricted to the interval determined in part (B).

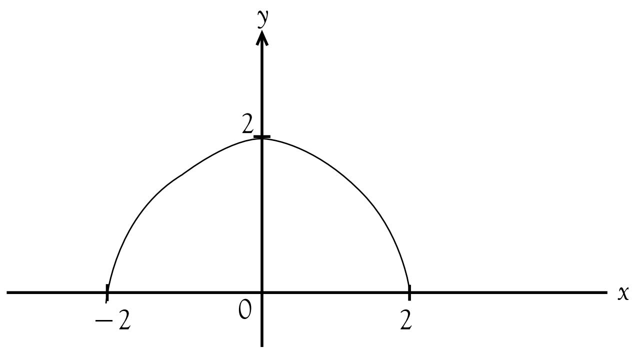
Sample Response That Received a Score of 5:

(A) $4 - x^2 \geq 0$

$x^2 \leq 4$

$-2 \leq x \leq 2$

(B)

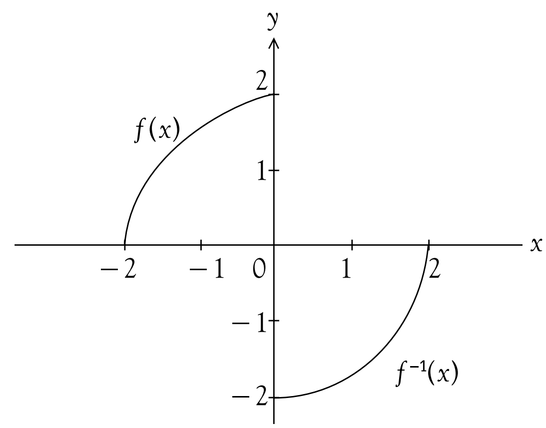


A graph of $f(x)$ is the figure above. A function that passes the horizontal line test has an inverse function. If one considers half of this semicircle, it will pass the horizontal line test. The half that contains $x = -1$ is the interval where $-2 \leq x \leq 0$.

(C) On the graph in part B the portion of the curve that contains $x = -1$ has the equation

$f(x) = +\sqrt{4 - x^2}$ where $-2 \leq x \leq 0$.
The inverse function has the equation

$f^{-1}(x) = -\sqrt{4 - x^2}$ for $0 \leq x \leq 2$



Sample Response That Received a Score of 2:

(A) $[-2,2]$

(B) $[-2,2]$

(C) $y = \sqrt{4 - x^2}$

$y^2 = 4 - x^2$

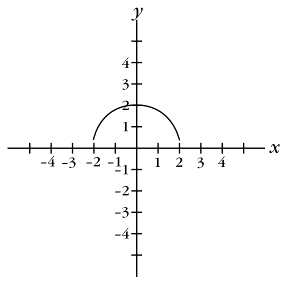
$y^2 - 4 = -x^2$

$x^2 = -y^2 + 4$

$x = \sqrt{4 - y^2}$

Sample Response That Received a Score of 1:

(A) On my graphing calculator, the function is graphed in the $[-5,5] \times [-5,5]$ viewing window and looks like



From the picture, I can see the domain of the function is $[-2, 2]$ and that the function has an asymptote at $x = 2$.

Sample Question 6: Problems

Let S be the set of all multiples of a positive integer k, where $k \geq 2$. An element $n \in S$ is called an S-prime if it cannot be written as a product of two or more elements of S. Any element of S that is not an S-prime can be factored in at least one way into a product of two or more S-primes.

For example, if S is the set of all positive multiples of 3, then 24 is an S-prime and 18 is not an S-prime $(18 = 3 \times 6)$.

Let S be the set of all positive multiples of 12.

(A) Find an element of S that can be factored into S-primes in two different ways.

(B) Exhibit (i) an S-prime with an integer square root and (ii) an S-prime with an integer cube root.

Sample Response That Received a Score of 5:

(A) $(6 \times 12) \times (6 \times 12) = (4 \times 12) \times (9 \times 12)$
$= 5184$

(B) (i) Want number $n \times 12$ where n is not a multiple of 12 and $\sqrt{n \times 12}$ is an integer.
$\sqrt{n \times 12} = \sqrt{2^2 \cdot 3 \cdot n}$. Let $n = 3$, so
$\sqrt{3 \times 12} = 6$ is an integer square root.
Therefore, $n \times 12 = 36$.

(ii) Want number $n \times 12$ where n is not a multiple of 12 and $\sqrt[3]{n \times 12}$ is an integer. If $\sqrt[3]{n \cdot 2^2 \cdot 3}$ is an integer, we need $n = 2 \cdot 3^2 = 18$. So
$\sqrt[3]{18 \cdot 2^3 \cdot 3} = \sqrt[3]{2 \cdot 3^2 \cdot 2^2 \cdot 3}$
$= \sqrt[3]{2^3 \cdot 3^3} = 6$
So $n \times 12 = 216$

Sample Response That Received a Score of 1:

(A) $S = \{12, 24, 36, 48, 60, 72, 84, 96, 108, 120, 132, 144, \ldots\}$
$(2 \times 12) \times (6 \times 12) = (3 \times 12) \times (4 \times 12)$
$= 1728$

(B) (i) 144 is a perfect square so it has an integer square root 12.
(ii) 1728 is a perfect cube so it has an integer cube root 12.

Sample Question 7: Problems

Consider the function f defined for all real numbers by $f(x) = \dfrac{x^4}{2} - 3x^3 - x^2 + 15x + c$, where c is a real-valued constant.

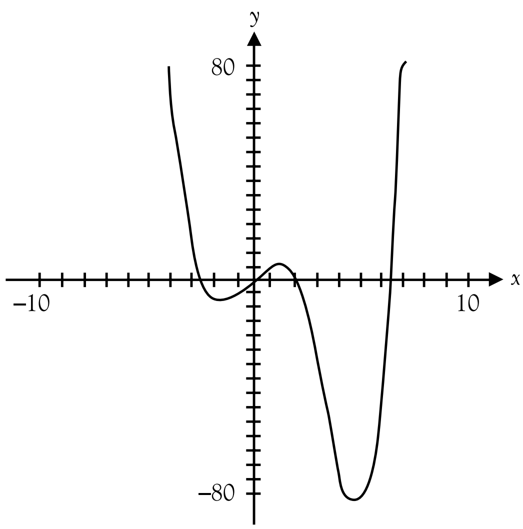
(A) Determine a value of c for which the function has exactly 4 real roots. Explain how you arrived at your answer.

(B) Determine a value of c for which the function has no real roots. Explain how you arrived at your answer.

(C) Determine a value of c for which the function has exactly 2 real roots. Explain how you arrived at your answer.

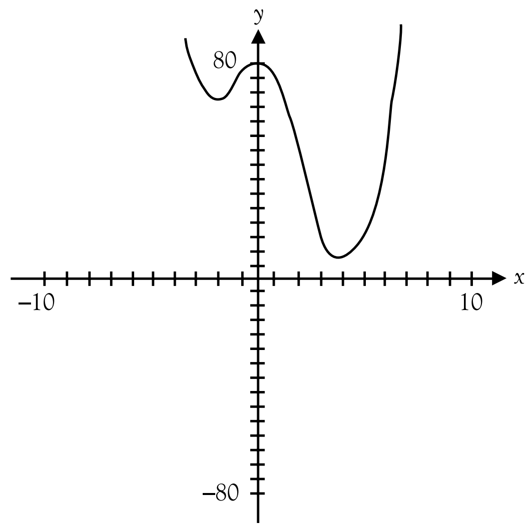
Sample Response That Received a Score of 5:

(A) If $c = 0$, the graph of f looks like this:

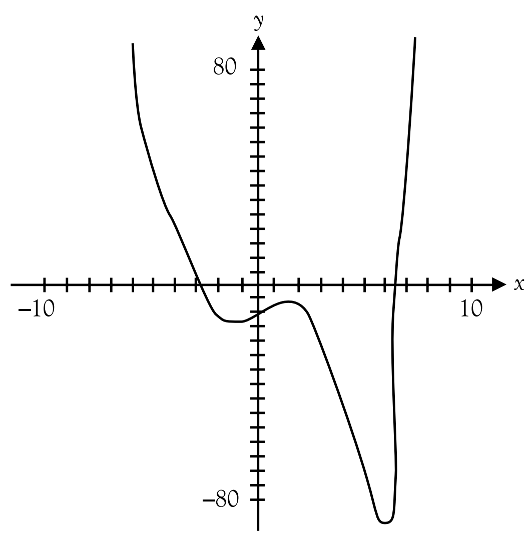


This graph has exactly four real roots because it crosses the x-axis at 4-points.

(B) For the graph to have no roots, it must be moved up enough so that its minimum is greater than 0. By tracing along the curve, I see that the minimum is about -75, so a value of $c = 80$ should do. Checking this on my calculator, I see that when $c = 80$, the graph of f has no roots. This is the graph on my calculator

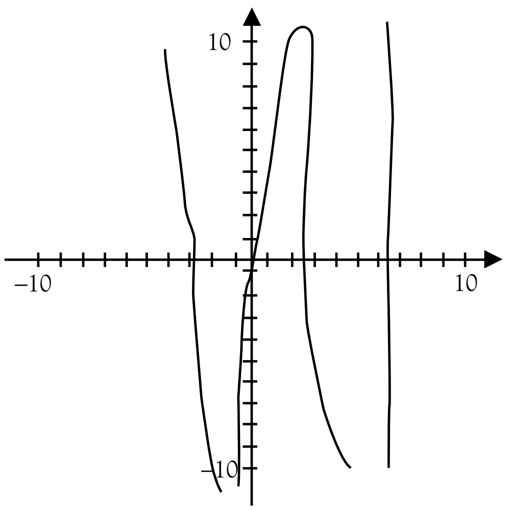


(C) For the graph to have exactly two roots it must be moved down enough so that the relative maximum at about $x = 1$ is negative. By tracing along the curve, I see that this relative maximum is about 10, so a value of $c = -15$ should do. Checking this on my calculator, I see that when $c = -15$, the graph of f has exactly two roots. This is the graph on my calculator



Sample Response That Received a Score of 2:

(A) $f(x)$ has exactly 4 real roots if the graph of $f(x)$ has exactly 4 x-intercepts.



This is the graph of $f(x)$ if $c = 0$.

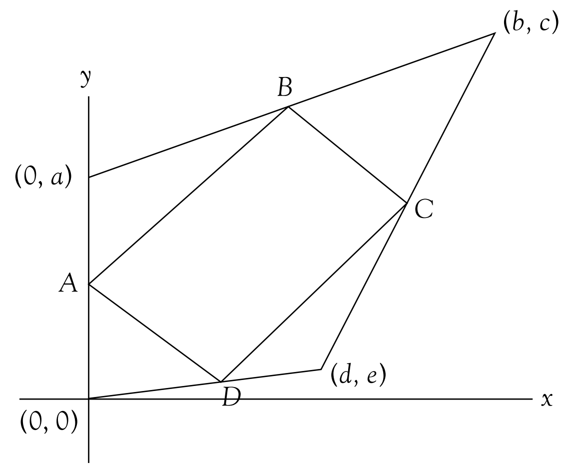
(B) $f(x)$ always has at least 2 roots.

(C) $f(x)$ has exactly 2 real roots if the graph of $f(x)$ has 2 x-intercepts.

Sample Question 8: Proof

Prove that the lines joining the midpoints of the adjacent sides of any quadrilateral form a parallelogram.

Sample Response That Received a Score of 5:



Place quadrilateral in coordinate plane so that the angle that is ≤ 90° is at origin and one side lies on the y-axis. Coordinates of vertices are shown.

Coordinates of midpoints

$A = \left(0, \frac{a}{2}\right)$ $\quad C = \left(\frac{b+d}{2}, \frac{c+e}{2}\right)$

$B = \left(\frac{b}{2}, \frac{a+c}{2}\right)$ $\quad D = \left(\frac{d}{2}, \frac{e}{2}\right)$

To show $ABCD$ is a parallelogram, show slope AB = slope CD and slope BC = slope AD.

$$\text{Slope } AB = \frac{\frac{a+c}{2} - \frac{a}{2}}{\frac{b}{2}} = \frac{c}{b}$$

equal

$$\text{Slope } CD = \frac{\frac{c+e}{2} - \frac{e}{2}}{\frac{b+d}{2} - \frac{d}{2}} = \frac{c}{b}$$

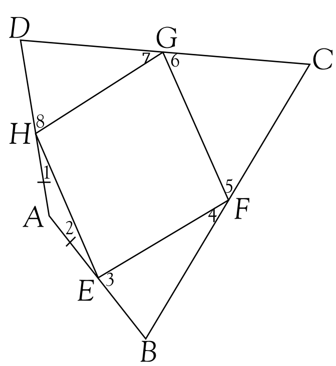
$$\text{Slope } BC = \frac{\frac{c+e}{2} - \frac{a+c}{2}}{\frac{b+d}{2} - \frac{b}{2}} = \frac{e-a}{d}$$

equal

$$\text{Slope } AD = \frac{\frac{e}{2} - \frac{a}{2}}{\frac{d}{2}} = \frac{e-a}{d}$$

Sample Response That Received a Score of 1:

Let $ABCD$ be a quadrilateral with
E the midpoint of AB
F the midpoint of BC
G the midpoint of CD
H the midpoint of AD



$\overline{AE} \cong \overline{AH}$
$\overline{BF} \cong \overline{FC}$ Def. midpoint
$\overline{CG} \cong \overline{GD}$
$\overline{DH} \cong \overline{AH}$

$\angle 1 \cong \angle 2$
$\angle 3 \cong \angle 4$
$\angle 5 \cong \angle 6$ Base $\angle$'s of isosceles Δ are $\cong$
$\angle 7 \cong \angle 8$

$\Delta DHG \cong \Delta BFE$ by ASA
$\overline{HG} \cong \overline{EF}$
$\overline{HG} \parallel \overline{EF}$
$EFGH$ is a quadrilateral with a pair of opposite sides $\cong$ and $\parallel$ $\therefore$ $EFGH$ is a parallelogram.

Sample Question 9: Proof

A function g defined for all real numbers is called an even function if $g(-x) = g(x)$ for all real x.

A function h defined for all real numbers is called an odd function if $h(-x) = -h(x)$ for all real x.

(A) If f is any function defined for all real numbers, prove that $f(x) + f(-x)$ is an even function.
(B) If f is any function defined for all real numbers, prove that $f(x) - f(-x)$ is an odd function.
(C) Prove that any function f defined for all real numbers can be written as the sum of an even function and an odd function.

Sample Response That Received a Score of 5:

(A) Prove $f(x) + f(-x)$ is an even function.
Let $g(x) = f(x) + f(-x)$
$\Rightarrow g(-x) = f(-x) + f(-(-x)) = f(-x) + f(x) = f(x) + f(-x) = g(x)$
$\Rightarrow g(-x) = g(x) \therefore f(x) + f(-x)$ is an even function.

(B) Prove $f(x) - f(-x)$ is an odd function.
Let $g(x) = f(x) - f(-x)$
$\Rightarrow g(-x) = f(-x) - f(-(-x)) = f(-x) - f(x) = -(f(x) - f(-x)) = -g(x)$
$\Rightarrow g(-x) = -g(x) \therefore f(x) - f(-x)$ is an odd function.

(C) Let $g(x) = \frac{1}{2}f(x) + \frac{1}{2}f(-x)$ an even function

Let $h(x) = \frac{1}{2}f(x) - \frac{1}{2}f(-x)$ an odd function

$g(x) + h(x) = \frac{1}{2}f(x) + \frac{1}{2}f(-x) + \frac{1}{2}f(x) - \frac{1}{2}f(-x) = f(x)$ for any function f defined for all $\boldsymbol{R}$.

Sample Response That Received a Score of 1:

(A) Suppose $f(x) + f(-x)$ is odd. Therefore $f(x) + f(-x) = f(x) - f(x) = 0$. This contradicts itself so $f(x) + f(-x)$ is even.

(B) Suppose $f(x) - f(-x)$ is even. Therefore $f(x) - f(x) = 0$. This contradicts itself so $f(x) - f(-x)$ is odd.

(C) Function $x + 1$

even $= x$ odd $= 1$

or

odd $= x$ even $= 1$

Sample Question 10: Modeling

School Year	Salary (top of scale)
1988-1989	$51,400
1989-1990	$55,300
1990-1991	$59,500
1991-1992	$64,000
1992-1993	$68,800

Under the teacher-pension plan in a certain state, the **basis** for the annual pension for a retired teacher is the average of the retired teacher's 3 highest annual salaries. The annual pension is computed when a teacher retires by taking 2% of the **basis** and multiplying it by the total number of years the teacher has taught in the state.

The table above shows the negotiated annual salary scale from 1988 to 1993 for teachers at the top of the scale in a school district. Prior to the 1988-1989 school year, the annual salary for a teacher at the top of the scale in the district was less than $51,400. Two teachers, Ms. Apple and Mr. Gonzales, have been at the top of the scale in the district for the past 10 years; neither has received any additional salary for extra activities.

(A) The 1990-1991 school year marked the end of Ms. Apple's 33rd year of teaching in the state. She is trying to decide whether she should teach another 2 years. How much more would her annual pension be if she retired at the end of the 1992-1993 school year instead of at the end of the 1990-1991 school year?

(B) Mr. Gonzales determined that his annual pension would be $38,460 if he retired at the end of the 1992-1993 school year. The 1993-1994 contract calls for a salary increase of k dollars at the top of the scale. If Mr. Gonzales decides to postpone his retirement until the end of the 1993-1994 school year, what would be the increase in his annual pension in terms of k?

Sample Response That Received a Score of 5:

(A) Basis $= \dfrac{\text{total high 3 years}}{3}$

Pension $= 2\% \times \text{basis} \times \text{years taught}$

$$\text{Basis}_{91} \ \frac{51,400 + 55,300 + 59,500}{3} = \frac{166,200}{3}$$

$= 55,400$

$\text{Years}_{91} = 33$

$$\text{Basis}_{93} \ \frac{59,500 + 64,000 + 68,800}{3} = \frac{192,300}{3}$$

$= 64,100$

$\text{Years}_{93} = 35$

55,400 = (avg) basis if 1990-91
64,100 = (avg) basis if 1992-93

Apple 1991 (33 yrs) $\rightarrow$ 36,564
1993 (35 yrs) $\rightarrow$ 44,870

$8,306 more if she retired at the end of 1992-1993 school year

(B) x = years Gonzales taught if 1993 last year
$38{,}460 = 64{,}100(0.02)x$
$x = 30$
38,460 if 92-93

Increase if he waits until end of 93-94

$$= \frac{201,600 + k}{3}(0.02)(31) - 38{,}460$$
$$= \frac{0.62}{3}(201{,}600 + k) - 38{,}460$$
$$= 41{,}664 + \frac{0.62k}{3} - 38{,}460$$
$$= \frac{0.62}{3}k + 3{,}204$$

Sample Response That Received a Score of 1:

(A) $(2\%)(59{,}500)(33) = 39{,}270$
$(2\%)(68{,}800)(35) = \underline{48{,}160}$

It would be $8,890 more dollars

(B) $38{,}460 = (.02)(68{,}800)$(number of years)
28 years

k = increase in dollars
Annual pension = $(.02)(29 \text{ years})(68{,}800 + k)$
Annual pension = $39{,}904 + (.58)(k)$

Sample Question 11: Modeling

A certain florist sells two special floral arrangements, regular and deluxe. These arrangements use three varieties of fresh flowers — carnations, roses, and tulips. The regular arrangement contains 2 carnations, 1 rose, and 2 tulips. The deluxe arrangement contains 4 carnations, 2 roses, and 3 tulips.

The arrangements are prepared at the beginning of the day and then stored under refrigeration for later sales to customers. For a certain holiday the florist will have available 160 carnations, 90 roses, and 140 tulips.

The florist will make a profit of $5.50 on each regular arrangement and $7.40 on each deluxe arrangement that is sold. How many of each type of floral arrangement should be prepared if profits are to be maximized on their sale and it is assumed that each arrangement that is made will be sold? What is the maximum profit?

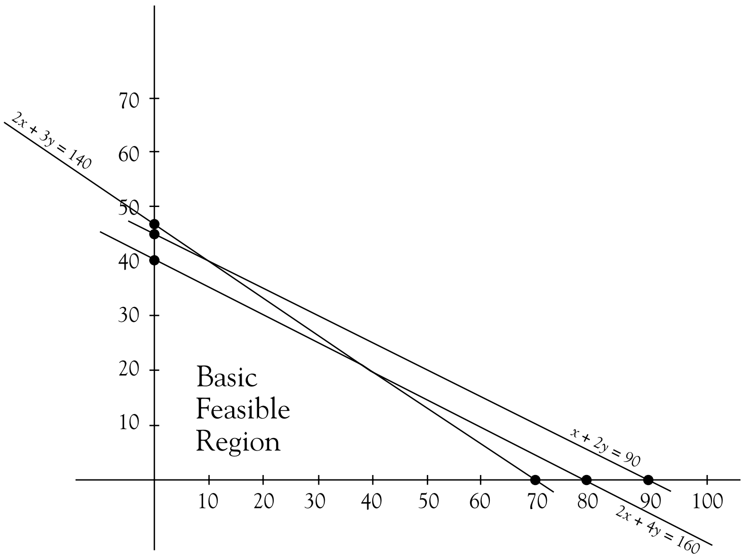
Briefly explain the mathematics involved in your solution.

Sample Response That Received a Score of 5:

	Number of Arrangements	Carnations	Roses	Tulips
Regular	x	$2x$	x	$2x$
Deluxe	y	$4y$	$2y$	$3y$
Total		160	90	140

Carnations: $2x + 4y \le 160$
Roses: $x + 2y \le 90$
Tulips: $2x + 3y \le 140$
where $x > 0$ and $y > 0$

Graph the inequalities to get the basic feasible region



Find the point of the basic feasible region that maximizes profit $P = 5.50x + 7.40y$

The corners of the B.F.R. are

(0, 0) $P = 0$
(0, 40) $P = 5.50(0) + 7.40(40) = \$296$
*(40, 20) $P = 5.50(40) + 7.40(20) =$
$220 + 148 = 368$
(70, 0) $P = 5.50(70) + 7.40(0) = 385$

The maximum profit of $385 occurs at the corner (70, 0). The maximum profit occurs when 70 regular and zero deluxe arrangements are made.

*Solve
$$\begin{aligned} 2x + 4y &= 160 \\ 2x + 3y &= 140 \\ \hline y &= 20 \\ 2x + 80 &= 160 \\ 2x &= 80 \\ x &= 40 \end{aligned}$$
(40, 20) is the corner

Sample Response That Received a Score of 1:

	Number of Arrangements	Carnations	Roses	Tulips
Regular	x	2	1	2
Deluxe	y	4	2	3

T. Carnation 160
T. Roses 90
T. Tulips 140

Profit $P = 5.50x + 7.40y$

Profit is a max. at (2, 4)
$5.50(2) + 7.40(4) = 11.00 + 29.60 = \$40.60$
Max. Profit is $40.60